STeady-state model of heterogeneous detonation with reactive metallic particles

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A comprehensive, 1-D model of the steady-state detonation of a condensed explosive with reactive particles was developed. The model accounts for carrier, particle and oxide phases, velocities and temperatures of the carrier and particle phases, their densities, pressure, etc. The features of the model are: sub-models for particle vaporisation and boiling at high pressures and particle combustion, a method of determining source terms for oxygen carriers in the flow of detonation products, a sub-model for the interaction between the particle and leading shock wave front and generalisation of the conditions at the leading shock wave. The results of calculations using this model have shown that a heterogeneous mixture of RDX with ultra-dispersed aluminum particles (0.1 µm) leads to a reduction in the detonation velocity compared to its corresponding value for pure RDX or a mixture with inert particles. The ultra-dispersed particles are burned completely in the RDX reaction zone for detonations with particle mass fraction less than 20%. The model results indicate that the detonation velocity of the heterogeneous mixture decreases to a greater extent as the intensity of the heterogeneous and vapour-phase combustion of the particles is increased. An important factor affecting the decrease in detonation velocity is the reduction in mass fraction of the gaseous component of the detonation products. This parameter is reduced, on average, by ~9 to 23% when the mass fraction of the metallic particles increases from 10 to 30%. Analysis of the model results for the detonation of RDX containing 5 µ-sized aluminum particles has shown that the particles do not burn in the reaction zone.

INTRODUCTION

This paper describes the development of a comprehensive model of a steady detonation wave in a heterogeneous charge with metallic reactive particles, its various sub-models, analysis of the numerical results for different RDX/Al mixtures and comparison with experiment. This model contains sub-models for the vaporisation of the metallic particles at high pressures, the vapour-phase and heterogeneous combustion of the particles,
and the particle interaction with the leading shock wave.

**BASIC MODEL FOR THE STEADY-STATE HETEROGENEOUS DETONATION WITH REACTIVE METALLIC PARTICLES**

The one-dimensional steady-state motion of a planar detonation wave in a continuous medium is considered in this model. The basic system of equations for multi-phase fluid dynamics, in the coordinate system fixed to the detonation wave, can be written in the following form,

\[
\begin{align*}
\frac{d\rho u_1}{dx} &= J_1, \\
\frac{d\rho u_2}{dx} &= J_2, \\
\frac{d\rho u_3}{dx} &= 0, \\
\rho \frac{du_1}{dx} &= \frac{dp}{dx} + J_3, \\
\rho \frac{du_2}{dx} &= J_4, \\
\frac{d}{dx}[\rho u_1(e_1 + \frac{u_1^2}{2}) + \rho u_2(e_2 + \frac{u_2^2}{2}) + pu_1] &= J_5, \\
\frac{d}{dx}(\rho u_2 e_2) &= J_6.
\end{align*}
\]

Indices "1" and "2" refer to the carrier and dispersed phases, respectively. The various \(J_i\) terms denote the source terms and, the rest of the terms such as \(u, p, e\) and \(\rho\) are standard terms denoting velocity, pressure, internal energy and partial density, respectively. The multi-phase, fluid dynamic equations [Eq. (1)] along with the equations-of-state (EOS) for the condensed explosive, detonation products and particles are needed to complete the basic model equations.

In the present case, Tait’s EOS is used for the undecomposed explosive and the particle interaction with the leading shock wave. The EOS suggested by Kuznetsov et al.\(^1\) has been used for the detonation products. The ZND model for the reaction zone of the primary explosive was modified to take into account the effect of the particles. This modification refers to the substitution of the leading shock wave front by a generalized surface of discontinuity.

It is believed that the flow of the detonation products strips the oxide film off the particle surface, as suggested by Ref. 2. Therefore, the carrier phase can be considered to be composed of three components: 1 - the undecomposed explosive and the gaseous products (denoted by their mass-fraction, \(c_e\)), 2 - the oxide particles stripped off the surface of the metallic particles (denoted by the mass fraction \(c_{ox}\)), and, 3 - the oxide particles formed due to the vapour-phase reaction of the metallic particles (denoted by the mass fraction, \(c_{ox}'\)). The values of \(c_e\), \(c_{ox}\) and \(c_{ox}'\) can be determined from the following diffusion equations,

\[
\frac{d[\rho c^i(u_i + w_i)]}{dx} = J_{j+6}, \quad j=1,2,3, \quad (2)
\]

where \(c^1 = c_e\), \(c^2 = c_{ox}\) and \(c^3 = c_{ox}'\). The values of \(w_i\) denote the diffusion velocities which are usually considerably lower than the corresponding convective velocity, \(u_1\), and will be disregarded.

The oxidizer-carriers are primarily \(\text{H}_2\text{O}, \text{CO}_2\) and \(\text{CO}\) (and seldom \(\text{O}_2\)), and their mass fractions, denoted by \(c_i\) \((i = 1, 2 \text{ and } 3)\), are obtained from the following equations,

\[
\frac{d[\rho_i c_i(u_i + w_i)]}{dx} = J_{j+9}, \quad i = 1,2,3. \quad (3)
\]

Thorough analysis showed that the total system of equations [Eqs. (1) - (3)] could not be closed, even if it was assumed that all the source terms \(J_i\) and EOS were known. It was therefore necessary to derive additional equations to close the above system of equations.

Considering a unit volume of a mixture containing undecomposed explosive, detonation products, and metal and oxide particles, the following additional equations can be derived,

\[
\rho = \rho' \frac{\alpha_1}{c_e}, \quad \alpha_1 = \alpha_1[1 - \rho(1 - c_e)/\rho c)], \quad (4)
\]

where \(\rho'\), \(\rho_c\) and \(\alpha_1\) are the true density of the detonation products (together with the undecomposed explosive), the true density of the oxide particle and the volume fraction of
the detonation products plus the 
undecomposed explosive and oxide particles, 
respectively. The term, \( \alpha_1 \), represents the 
volume fraction of the detonation products 
plus the undecomposed explosive. Therefore, 
Eqs. (1)-(4), along with the EOS and the 
source terms, form the closed system of 
equations.

### SUB-MODEL FOR PARTICLE 
VAPOURISATION AND BOILING AT 
HIGH PRESSURES

The combustion of a particle in the reaction 
and expansion zones of the detonation wave of 
a primary explosive occurs with a high 
external pressure, \( p_g \) (\( \sim 10^{10} \) Pa). In this case, 
the vapour-phase fraction at the surface of the 
metal particles (Al, Mg), calculated using the 
well-known Clausius-Clapeyron equation,

\[
p_v^0 = \rho^* e^{-l/(RT)},
\]

was found to be negligible even at 
temperatures of several thousand degrees.

The derivation of the corresponding 
equation, taking into account the effect of 
external pressure, was presented in Ref 3.

These results\(^3\) are necessary for the 
development of the particle combustion model 
presented below. A generalized Clausius-
Clapeyron equation was obtained in the 
following form,

\[
\frac{P_v}{\rho} \exp\left(\frac{\Delta H_e}{R T}\right) = \exp\left[-\frac{(1-\frac{\mu_m}{\rho}(p^*-p_v^0))}{(RT)}\right],
\]

where \( p_v^0 \) represents the partial vapour 
pressure, defined by Eq. (5), \( \mu_m \) is the 
molecular mass of the vapour, and \( \rho \) is the 
density of the condensed material.

The dependence of the boiling point, \( T_b \), 
on the pressure reduces to the following 
explicit equation,

\[
T_b = \frac{1-(2\mu_m p_g/\rho)}{R \log(p^*/p_g)},
\]

It is interesting to note that the second term in 
the numerator of Eq. (7) is missing in 
Clausius-Clapeyron approach. Therefore, the 
values of \( T_b \) predicted by Eq. (7) will be lower 
than the corresponding values obtained based 
on the Clausius-Clapeyron equation\(^4\).

### ASYMPTOTIC SUB-MODEL FOR THE 
VAPOUR-PHASE 
AND HETEROGENEOUS COMBUSTION OF 
PARTICLES

The physical model for the combustion of a 
particle, presented below, is based on the 
assumption\(^5\) that heat and mass transfer 
between the particle and medium takes place 
inside a diffusion layer, with radius \( r_1 \), 
determined by the radius of the particle, \( r_0 = 
d/2 \), and the Nusselt number, in the form 
r_1 = r_0 Nu/(Nu-2).

It is assumed that both the heterogeneous 
and the vapour-phase reactions are first order 
with respect to the oxygen carriers and obey 
the Arrhenius temperature law,

\[
k_{oi} = k_0^* \rho_1^{02} n_i^0 \exp(-E_i/RT_2), \\
k_i = k_i^* (\rho_1^0)^2 n_i n_v \exp(-E_i/RT_2),
\]

where \( k_{oi} \) and \( k_i \) are the rates with respect to 
the metal, \( k_0^* \) and \( k_i^* \) are the reaction constants, 
\( n_i^0 \) and \( \rho_1^0 \) are the mass fractions of the oxygen 
carriers at the particle surface and density of 
the carrier phase in the vicinity of the particle 
surface, respectively, and \( n_i \) and \( n_v \) are the mass fractions of the oxygen 
carriers and metal vapour within the diffusion 
layer.

According to Ref. [6], a new, independent 
variable is introduced as,

\[
z = \exp[\rho_1^{02} v_0 r_0 c_p \lambda_0^0 (1-r_0/r)] - 1, \quad r_0 \leq r \leq r_1
\]

where \( \lambda_0 \) and \( c_p \) are the thermal conductivity 
and specific heat, respectively.

An approximate solution to the standard 
system of heat transfer and diffusion equations 
can be found using the following series 
expansion: \( T = T_0 + \varepsilon T_1 + \ldots \), \( n_i = n_i^0 + \varepsilon n_i^1 + \ldots \), where \( \varepsilon = n_i^0 = \mu_n [\mu_n(1 + \frac{p_g}{p_v})] \) (< 1) is 
a small parameter, and \( n_i^0 \) is the vapour mass 
fraction at the particle surface, determined
using Eq.(6), and, \(\mu\) is the molecular mass of the surrounding gas.

A series of calculations was carried out resulting in the following correlation for \(n^0_i\):

\[
n^0_i = \frac{c_i(1-Q_i)}{1 + z_i(1 + D_i)},
\]

\[
Q_i = \frac{dn^0_i \rho^0_i (k_i^* / k_{io}^*) \exp\left[-(E_i - E_i^*) / RT_i\right]}{Nu z_i (1 + \Pi_i) \log(1 + z_i)},
\]

\[
D_i = \frac{c_p d \rho^0_i \beta_i k_i^* \exp\left(-E_i^* / RT_i\right)}{\lambda_0 Nu \log(1 + z_i)}.
\]

where, \(Q_i \leq 1\), \(\Pi_i D_i = (1 + z_i) / z_i\), \(z_i = z(r_i)\) and \(\beta_i\) is the stoichiometric ratio. The term \(P_i D_i = P_i(z_i)\), where \(P_i(z)\) is a given polynomial. For values of \(Q_i\) in Eq. (10) greater than 1, the equality \(Q_i = 1\) is used. Finally, the function \(z_i(x)\) is found using the following equation,

\[
\sum_{i=1}^{N} \frac{Q_i}{1+\Pi_i} - 1 - Q_i - n^0_i(1 + z_i) - z_i = 0,
\]

(11)

The parameters, \(E_i^0\) and \(E_i\), in Eq. (10), are activation energies for heterogeneous and vapour-phase reactions, respectively. In the calculations below, it is assumed \(E_i^0 = E_i\). It is important to note that the value of the ratio \(k_i^* / k_{io}^*\), the ratio of the rate constants of the vapour-phase reaction to the corresponding value for the heterogeneous reaction of the metallic particle, in Eq. (10) are unknown. In order to overcome this limitation, it is assumed that only the vapour-phase combustion of the particle occurs at its boiling point, therefore, \(Q_i = 1\) since \(n^0_i = 0\). This assumption facilitates the exclusion of the ratio \(k_i^* / k_{io}^*\) from Eq. (10).

Therefore, the sub-model for particle combustion reduces to Eq. (11) with respect to the function \(z_i(x)\), which is solved simultaneously with the general system of equations (1) – (3).

**SUB-MODEL FOR PARTICLE INTERACTION WITH THE LEADING SHOCK FRONT OF A DETONATION**

To estimate the particle and flow parameters behind the leading shock wave, a 1-D method of the break-up of the arbitrary discontinuity and (mass, momentum and energy) balance equations at the shock wave was used. Overall, nine equations were obtained for nine unknown parameters, namely, \(u_i, \rho_i, T_i, (i = 1, 2)\), \(p\), \(n\) and \(\alpha_i\), which allowed the calculation of their dependencies on the detonation velocity, \(D\), and particle mass fraction, \(m_2\). Calculations of all carrier phase and particle parameters behind the leading shock wave were carried out, in the system of coordinates attached to the shock wave, with the following parameters, \(4000 \leq D \leq 10000\) m/s, \(0 \leq m_2 \leq 0.4\).

It appears the detonation velocity is very sensitive to values of the output parameters that are the boundary conditions for the system of equations (1). As a result, the numerical 2-D modeling of the shock wave diffraction over a spherical compressible particle (Al and Mg) in RDX, with the HOM EOS was produced. The results were compared with data from the 1-D approach. The values for particle temperature, \(T_2^p\), are shown in Fig. 1 by the solid circles, corresponding to the 2-D numerical method for RDX (\(\rho = 1.4\) g/cm\(^3\)) with Al particles. The solid line, obtained from 1-D approach, displays good agreement with the numerical modeling.

A comparison of the ratio of the absolute particle velocity with the flow velocity, obtained with the 2-D numerical method and using modified the 1-D approach, yields comparable results as well.

**VARIOUS SOURCE TERMS AND CORRELATIONS REQUIRED TO OBTAIN A CLOSED SOLUTION**

The mass fluxes between the carrier and dispersed phases, denoted by \(J_1\) and \(J_2\), respectively, are related by the condition \(J_1 = -J_2\). The mass flux consists of three components, 1) the flux of the oxides formed
due to the heterogeneous combustion of the metal particles, denoted by $J_c$, 2) the flux of the metal vapour formed due to its evaporation, denoted by $J_m$, and, 3) the flux of oxygen into the dispersed phase, denoted by $J_0$. In the case of a stoichiometric reaction between the metallic particle and the oxidizer, the flux $J_0$ can be written as,

$$J_0 = \nu_0 J_c, \text{ for } T_0 \leq T_2 \leq T_h$$  \hspace{1cm} (12)

The mass flux of the metal vapour due to evaporation from the surface of the particle, $J_m$, can be determined by the following equation,

$$J_m = 6\alpha_2 \beta \sum_{i=1}^{N} \beta_i k_i n_i \exp(-E_0^i / RT_2) / d$$ \hspace{1cm} (13)

The Nusselt number is given by a well-known expression, as a function of Mach, Reynolds and Prandtl numbers. Assuming the rates of the heterogeneous reactions are represented by the formulae given in Eq. (8), the source term $J_c$ can be represented by the following equation,

$$J_c = 6\alpha_2 \nu_c \beta \sum_{i=1}^{N} \beta_i n_i \exp(-E_0^i / RT_2) / d$$ \hspace{1cm} (14)

where $\nu_c$ denotes the stoichiometric ratio for the heterogeneous combustion of Al to Al$_2$O$_3$.

The next step is to determine the forces of interaction between the interfaces, defined by the momentum flux terms, $J_3$ and $J_4$. The drag coefficient is determined by the following correlation, proposed by Henderson,

$$C_D = (24/Re) L_D$$  \hspace{1cm} (15)

Besides the source terms, $J_1$-$J_6$, the system of model equations, given by Eqs. (2) and (3), also contains six other unknown source terms. It can be stated that $J_8=J_c$ and $J_9=\nu_c J_m$. The source term, $J_9$, is developed based on the assumption that the total conversion of all the metal vapour into the oxide is accomplished. Therefore, from Eq. (2), the source term, $J_7=(1-\nu_c)J_1$.

The rate of decomposition of the primary HE is assumed to be represented by the Arrhenius rate law in terms of the following correlation,
\[ u_d(c, Z) / dx = -c_e Z [a \exp(-E / RT_e) + J_1 / \rho_1] \]  

(19)

For a pure explosive, \( c_e = 1 \) and \( J_1 = 0 \); and Eq. (19) reduces to the well-known form of the Arrhenius rate law.

The remaining source terms, \( J_{10} - J_{12} \), have been determined based on the following hypothesis. For a primary explosive, represented by \( \text{CaHbNcOd} \), the overall detonation reaction at the C-J point can be represented by the following chemical reaction\(^{10}\),

\[
C_\sigma H_\rho N_\omega O_d = \gamma_1 H_2O + \gamma_2 CO_2 + \gamma_3 CO + \gamma_4 O_2 + \ldots + C_{(a)},
\]

(20)

that leads to the following equality,

\[
\sum_{i=1}^{N} \frac{\gamma_i \mu_i}{\mu_e} = 1,
\]

where \( \mu_e \) and \( \mu_i \) denote the molecular masses of the primary explosive and the detonation products (such as \( H_2O, CO_2, CO \) etc.), and \( \gamma_i \) denotes the number of moles of the corresponding product component, as given in Eq. (20). Every term on the left side of this equality represents the mass fraction of the corresponding species of the detonation products. It is assumed that these values for the mass fraction can be used at various intermediate points in the reaction zone. As a result, the source terms for various oxygen carriers can be represented by the following equation,

\[
J_i = c_e Z (\gamma_i \mu_i / \mu_e) \exp(-E / RT_e) + 6 \sigma \lambda_d \log(1 + z_i) \times (c_e - (1 + z_i) (m_1 / \rho_1)) / (c_e d^2), \quad i = 1, 2, 3
\]

(21)

The first term in Eq. (21) represents the flux of the oxygen carrier, produced by the decomposition reaction of the primary explosive, and the second term represents its consumption due to the heterogeneous and vapour-phase reactions between the oxygen carrier and the metal particle.

The "shooting" method, using the solutions for different values of the detonation velocity, was implemented to obtain the true solution.

MODEL RESULTS AND THEIR ANALYSIS FOR HETEROGENEOUS DETONATIONS WITH RDX CONTAINING ULTRADISPERSED ALUMINUM PARTICLES

As a first effort, a series of runs was conducted to predict profiles of various parameters for detonations with medium-density (1.4 g/cm\(^3\)) RDX containing 0.1-μm diameter Al particles in mass fractions (\( m_2 \)) ranging from 0.1 to 0.3. The ranges of the values for various rate constants, \( k^*_o \), were selected as following:

- \( H_2O: k^*_{o1} = 14 \times 10^7 \div 140 \text{m/s} \)
- \( CO: k^*_{o2} = 4.1 \times 10^7 \div 41 \text{m/s} \)
- \( CO: k^*_{o3} = 5 \times 10^7 \div 5 \text{m/s} \)

An initial set of runs with \( m_2 = 0.1 \) indicated that for the maximum values of the rate constants, \( k^*_o \), shown above, a steady-state detonation did not exist. To obtain a steady-state solution, the values of \( k^*_o \) were reduced to one-tenth and one hundredth of their maximum value. The results from the reduced values of the parameters exhibited a steady detonation wave with a VOD of 6928 m/s for the minimum values of the rate constants. It is interesting to note that a decrease in the VOD of \( \sim 500 \text{ m/s} \) as the particle mass fraction increased from 0 to 10%, corresponds, qualitatively, relatively well with some of the experimental data reported in Refs. [11 and 12]. Note the VOD continuously decreases from 7429 m/s to 6203 m/s as the particle mass concentration is increased from 0 to 30%.

Profiles of the temperatures of the carrier and dispersed phases, \( T_1 \) and \( T_2 \) respectively, are given in Figs. 2 and 3. The flow temperature, \( T_1 \), increases sharply close to the leading shock wave and then decreases, particularly in cases of mixtures with higher Al concentrations (e.g. \( m_2 = 0.2 \) and 0.3). Subsequently, \( T_1 \) increases to its maximum value at the point where the decomposition of the primary explosive is complete. It should be noted that the flow temperatures in the front part of the reaction zone for detonations of Al mixtures are significantly lower than the corresponding value for pure RDX, and the difference increases with increasing mass.
concentration of the particles. However, the values of $T_1$ throughout the reaction zone are significantly greater for detonations with mixtures than corresponding values for pure RDX, which contributes additional explosive energy. The distribution of the particle temperature, $T_2$, as shown in Fig. 3, indicates that the particles are heated very rapidly. The temperature of the particles does not reach the boiling point and begins to decrease after reaching a maximum, as the particle size is reduced to zero.

The addition of the particles also results in a significant change in the pressure profile within the reaction zone. Instead of a gradual decrease in pressure, as observed for pure RDX (Fig. 4), it can be observed that there is a sharp increase in pressure close to the leading shock wave front, followed by a region of fairly gradual decrease, to a point where there is a sharp decrease in the pressure, followed by another region of fairly gradual decrease in the expansion zone. Overall, the maximum values of pressure and partial density are less for RDX/Al mixtures than those for pure RDX.

The profiles of $c_e$ within the reaction zone, as shown in Fig. 5, indicate a significant reduction in the gaseous component of the detonation products. For example, the average $c_e$ in the reaction zone is predicted to decrease by ~9%, 19% and 23% as the mass concentration of particles in the HE is increased to 10%, 20% and 30%, respectively. It is believed that a significant reduction in the concentration of $(c_\rho p_1)$, along with the energy and momentum losses due to the particles, are the primary factors causing the decrease in detonation velocity due to the addition of the reactive metallic particles. It is interesting to note that the experimental results in Ref. [11], allowed its author to conclude that the decrease in VOD from the detonation of pure explosive, compared to one involving mixtures with reactive particles, was greater than the corresponding decrease in detonations involving mixtures with inert particles. This behaviour is understandable if the reduction in the gaseous concentration of the carrier phase in the case of reactive particles is considered, whereas, a similar reduction does not occur in the case of inert particles.

The distribution of the mass fraction of oxide particles, $c_{ox}$, produced by vapour-phase combustion is shown in Fig. 6. For $m_2=0.1$, 0.2 and 0.3, the mass fractions of particles burned in the vapour-phase reaction, were calculated to be ~6.3%, 4.4% and 3.8%, respectively. This indicates that the vapour-phase combustion of the particle at high pressures within the reaction zone is not very effective.

It should be noted that the important part of the heterogeneous detonation process is the mass fraction of oxygen-carriers in the detonation products, $c_i$, namely, H$_2$O, CO$_2$ and CO. The distribution of $c_1$ (H$_2$O) is shown in Fig. 7. In accordance with the total decomposition of the primary explosive (RDX), the values of mass fraction, $c_1$, as expected, reach a maximum, and then, particularly when $m_2 > 0.15$, the mass fraction of the oxygen-carrier starts decreasing, signifying the consumption of this carrier due to the fact that the combustion of the particles exceeds its production by the decomposition of RDX.

The distribution of the mass fraction of the oxygen carrier, H$_2$O, at the surface of the particle, denoted by $n_1^0$, is shown in Fig. 8. This distribution clearly indicates the combustion reaction at the particle surface occurs with mass fractions of the oxygen-carrier (H$_2$O) significantly lower than corresponding values in the detonation products.

The variation in particle size in terms of its diameter, $d_p$, within the reaction zone is shown in Fig. 9. It is important to note that for $m_2<0.2$, the particles are burned completely within the reaction zone. In the case of $m_2 = 0.25$, the combustion of the particles is not
completed within the reaction zone.

**MODEL RESULTS FOR THE HETEROGENEOUS DETONATION OF RDX/AL MIXTURES CONTAINING MICRON-SIZED REACTIVE PARTICLES**

The profiles of various parameters within the reaction zone were calculated for the detonation of moderate density (1.4 and 1.56 g/cm³) RDX explosives containing 5-µm diameter aluminum particles. The results from the model for the detonation of a RDX/Al mixture containing a 10% mass fraction of Al particles ($m_2=0.1$), in terms of the distribution of all flow and particle parameters were obtained.

The distribution of the particle velocity, $u_2$, and flow velocity, $u_1$, shows that $u_2$ in the reaction zone is greater than $u_1$, and the difference between them reduces to almost zero (~38 m/s) near the C-J plane. The pressure distribution for this example, as shown in Fig. 10, is similar to that obtained for pure RDX. However, over a large area of the reaction zone, the pressure, for detonations with reactive particles, is lower than the corresponding value for detonations of pure explosive due to losses to the particles.

The flow temperature with reactive particles is 600-800 K higher than that for pure RDX. This increase in temperature can be attributed to two factors namely, a slightly higher value for VOD and the retardation of the particles, resulting in the heating of the flow phase. It is to be note that the particle temperature reaches its melting point but the melting process is not complete within the reaction zone.

For detonations with mixtures having 5-µm diameter Al particles, the particle diameter remains the same throughout the entire reaction zone. From results calculated with this model, it can be concluded that the 5-µm diameter particles behave as inert particles in the reaction zone. The small increase in VOD observed for detonations of RDX containing 5-µm diameter reactive Al particles, compared to the corresponding value for pure RDX, as predicted by this model, can be explained with the same reasoning used to describe similar results for inert particles. It is important to note that according to Ref. [13], depending on the HE and the equation-of-state (EOS), the exact opposite behaviour, i.e. a decrease in VOD with the addition of 5-µm diameter particles, could also be obtained.

**CONCLUSIONS**

To summarize, it can be concluded that the steady-state heterogeneous detonation model results are viable; and it has been convincingly demonstrated that the increase in VOD for explosive mixtures involving the combustion of metallic particles is negligible, or almost impossible, and, in the majority of cases, a decrease in VOD occurred. Two factors in this model have the greatest impact on the values of VOD and the structure of the reaction zone. The first factor is the values of the initial flow parameters (temperature, density, etc.) behind the leading shock wave, which depends on particle mass fraction and EOS of the condensed HE. The second is the values of rate constants for heterogeneous and vapour-phase reactions of Al particles with the oxygen carriers.

The primary effect of the heterogeneous explosion is amplification of the blast wave, as a result of a higher flow temperature compared with the explosion of pure HE. Analysis of the results from a series of experimental tests has shown a definite decrease in VOD for mixtures containing RDX with ultradispersed particles and a slight decrease in the VOD for mixtures containing (5 µm) micron–sized particles. These results are in agreement with the predictions obtained from this model.
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REFERENCES


FIGURE 1. $T_2^1$ AS A FUNCTION OF D

FIGURE 2. $T_1$-DISTRIBUTION FOR RDX/AL MIXTURES (1.4 g/cm$^3$, 0.1 µm)
FIGURE 3. T2-DISTRIBUTION FOR RDX/AL MIXTURES (1.4 g/cm³, 0.1 µm)

FIGURE 4. P-DISTRIBUTION FOR RDX/AL MIXTURES (1.4 g/cm³, 0.1 µm)
FIGURE 5. C_r-DISTRIBUTION FOR RDX/AL MIXTURES (1.4 g/cm³, 0.1 μm)

FIGURE 6. C'ox-DISTRIBUTION FOR RDX/AL MIXTURES (1.4 g/cm³, 0.1 μm)
FIGURE 7. C₁-DISTRIBUTION FOR RDX/AL MIXTURES (1.4 g/cm³, 0.1 µm)

FIGURE 8. n₁₀-DISTRIBUTION FOR RDX/AL MIXTURES (1.4 g/cm³, 0.1 µm)
FIGURE 9. $d_p$ - DISTRIBUTION FOR RDX/AL MIXTURES (1.4 g/cm³, 0.1 µm)

FIGURE 10. $P$ - DISTRIBUTION FOR RDX/AL MIXTURES (1.4 g/cm³, 5.0 µm)
EQUATIONS

\[
\frac{d\rho u_1}{dx} = J_1, \quad \frac{d\rho u_2}{dx} = J_2, \quad \frac{dn_u}{dx} = 0, \\
\rho u_i \frac{du_i}{dx} = -\frac{dp}{dx} + J_3, \quad \rho u_2 \frac{du_2}{dx} = J_4, \\
\frac{d}{dx} \left[ \rho u_1 (e_1 + \frac{u_1^2}{2}) + \rho u_2 (e_2 + \frac{u_2^2}{2}) + pu_1 \right] = J_5, \\
\frac{d}{dx} (\rho u_2 e_2) = J_6.
\]

\[
\frac{d[\rho c^j (u_i + w_i)]}{dx} = J_{j+i}, \quad j = 1,2,3,
\]

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\]

\[
\rho_i = \rho^0 \frac{\alpha_i}{c_e}, \quad \alpha_i^1 = \alpha_i \left[ 1 - \rho_i (1 - c_e) / (\alpha_i \rho_i) \right],
\]

\[
\rho^0_v = p^* \exp(-l / RT),
\]

\[
\frac{P_p}{P} \exp\left( \frac{H_p p_s}{RT_p} \right) = \exp\left[ -\left( \frac{H_p}{\rho} (p_p - p^0) \right) / (RT) \right],
\]

\[
T_b = \frac{l - (2 \mu m p_g / \rho)}{R \log(p^*/p_g)}.
\]

\[
k_{0i} = k_{0i} \rho^0_v \exp(-E^0_i / RT), \\
k_i = k_i \left( \rho^0 \right)^2 n_i n_v \exp(-E_i / RT),
\]

\[
z = \exp[\rho^0 v_{0i} c_p \beta_i^2 (1 - r_0 / r)] - 1, \quad r_0 \leq r \leq r_i,
\]

\[
n_i^0 = \frac{c_i (1 - Q_i)}{1 + z_i (1 + D_i)}, \\
Q_i = \frac{d_i^0 \beta_{k_{0i}}}{\lambda_{0i} Nu \log(1 + z_i)} \exp\left[ (E_i - E_i^0) / RT \right],
\]

\[
D_i = \frac{c_i d_i^0 \beta_{k_{0i}}}{\lambda_{0i} Nu \log(1 + z_i)} \exp(-E_i / RT),
\]

\[
\frac{\sum_{i=1}^{N} \beta_i}{1 + \Pi_i} \left[ c_i + n_i^0 (1 + z_i) - z_i \right] = 0,
\]
\[ J_1 = J_m + (1 - \nu_0)J_c, \text{ for } T_0 \leq T_2 \leq T_h \]
\[ J_m = 6\alpha_2[\lambda_0 \text{NuLog}(1 + z_1)](c_p d) \]
+ \[ \rho_1^{\alpha_2} \sum_{i=1}^{N} \beta k_0^i n_0^i \exp(-E_0^i / RT_2)] / d \]

\[ J_c = 6\alpha_2 \nu_c \rho_1^{\alpha_2} \sum_{i=1}^{N} k_0^i n_0^i \exp(-E_0^i / RT_2)] / d, \]

\[ J_2 = -\rho_2 \rho_3 \sum_{i=1}^{3} \frac{3}{2} C_f(u - u)u - u d \frac{du}{dx} + 3\frac{du}{dx}(u - u)J, \]

\[ J_4 = -J_3 - (u_1 - u_2)J_1. \]

\[ e_1 = c_e(e + ZQ) + (1 - c_e)e_c - c_\alpha q_c, \]

\[ e_2 = \alpha T_m, \quad T_0 \leq T_2 \leq T_m \text{ or } \alpha T_m + wL, \quad T_2 = T_m \text{ or } \alpha T_m + L, \quad T_m \leq T_2 \leq T_{2g}. \]

\[ J_5 = 6\alpha_2 \sigma(1 - T_2^4) / d + J_c q_c, \]

\[ J_6 = 6\alpha_2[\lambda_0 \text{NuLog}(1 + z_1)](T_1 - T_2 - (q/c_p)u_0(1 + z_1) / z_1 + \]
+ \[ \rho_2^{\alpha_2} \sum_{i=1}^{3} \beta k_0^i n_0^i \exp(-E_0^i / RT_2)] / d^2 + \]

\[ J_4(q - e_2^i + \nu q^i) - J_m(e_2^i + l) \]

\[ u_1 d(c_e Z) / dx = -c_e Z[a \exp(-E / RT_1) + J_1 / \rho_1], \]

\[ C_1 H_2 N_2 O_4 = \gamma_1 H_2 O + \gamma_2 C O + \gamma_3 C O + \gamma_4 O_2 + ... + C_{(4)}, \]

\[ \sum_{i=1}^{N} \gamma_i \mu_i / \mu_c = 1, \]

\[ J_{i,0} = c_i \rho Z(\gamma_i \mu_i / \mu_c) a \exp(-E / RT_1) + 6\alpha_2 \lambda_0 \text{NuLog}(1 + z_1) \]
\times \[ (c_i - (1 + z_1))(d n / d \delta_2) / (c_p d^2), \quad i = 1, 2, 3 \]

\[ H_2 O: k_0 = 14 \times 10^3 \div 140 m/s, \quad CO_2: k_0 = 4.1 \times 10^3 \div 41 m/s, \]
\[ CO: k_0 = 5 \times 10^3 \div 5 m/s. \]