

# THE LEVEL SET METHOD APPLIED TO THREE-DIMENSIONAL DETONATION WAVE PROPAGATION

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On the basis of previous works on 2D detonation propagation, the code LS3D employing the level set method to calculate 3D detonation wave propagation has developed in this paper. With this code some propagation processes of detonation waves in HEs without boundaries and in different shaped IHE JB9014 charges, both initiated in different ways, have been studied, including multi-point initiation, rate stick, plane wave lens and two detonation wave interaction. The calculated results for JB9014 explosive agree well with the experimental results.

## 1 INTRODUCTION

The level set (LS) method developed by Osher and Sethian et al [1~2] describes the evolution of interface that propagate especially according to the dependence of their own curvature, for example the detonation waves in explosives. In LS method, a Hamilton-Jacobi called evolution equation here and describing the interface movements should be introduced from hydrodynamics or experimental research. The numerical solutions of the evolution equation under a set of boundary and initial conditions are performed with the LS codes to calculate the level-set functions. Then the interfaces are determined with a series of special values of the level-set function. The kind of method handles

automatically any front shape and any topological changes in explosive charges.

The physical foundation of the LS method is the same for the DSD (detonation Shock Dynamics) theory, i.e., the detonation front velocity is curvature-dependent. Then a geometrical model of detonation propagation can be proposed with a form similar to the eikonal equation in geometrical optics. However, the eikonal equation is of high order and high non-linear and very difficult to be solved numerically. In the 2D case without interaction between adjacent segments of the detonation front, the geometrical approach is considerably effective, since the front at next time step can be described as the envelope of the wavelets originating from the front at last time step [3]. Meanwhile the mechanics

boundary conditions in 2D case are easily matched for the above geometrically determined detonation front. Nevertheless, it is very difficult to extend the geometrical approach to the 3D case. We have to ask for the LS method which is an effective treatment for complicated interfaces and noted for its simple logic and programming.

Aslam and Bdzil [4] have applied this method to modeling detonation shock dynamics (DSD). They have developed a numerical method to implement arbitrarily complex 2D boundary conditions. Based on their works, we apply the LS method to 3D detonation wave propagation in explosive, and compile the calculation code LS3D. The code LS3D can be used in calculating the detonation wave propagation in explosive charges initiated in different way. It can deal with different shaped and component multi-medium explosive charges. Employing this code, we simulate some detonation propagation problems numerically in this paper.

## 2 THE LEVEL-SET EQUATION OF THREE-DIMENSIONAL DETONATION WAVE PROPAGATION

Considering the detonation wave propagating in 3D space of explosives, it is assumed that the detonation front is infinitely thin. In the space a rectangular coordinate system  $(x, y, z)$  has been defined, where this detonation front is described as the zero level set of a function  $\varphi(x, y, z; t)$ . Let a level set of points be defined by

$$\varphi(x, y, z, t = 0) = d \quad (1)$$

where  $d$  is the distance from point  $(x, y, z)$  to the detonation front. The set is corresponding to a contour surface. Particularly the detonation front is assigned by the level set  $\varphi = 0$ , while the unburnt explosive is denoted by  $\varphi > 0$  and the burnt explosive  $\varphi < 0$ .

Since a level surface is given by  $\varphi(x, y, z, t) = \text{const}$ , it follows that its total derivatives with respect to  $t$  identically equals zero, i.e.,

$$\frac{\partial \varphi}{\partial t} + \frac{\partial \varphi}{\partial x} \frac{dx}{dt} + \frac{\partial \varphi}{\partial y} \frac{dy}{dt} + \frac{\partial \varphi}{\partial z} \frac{dz}{dt} = 0 \quad (2)$$

where the time derivative  $dx/dt$ ,  $dy/dt$  and  $dz/dt$  are the components of the surface velocity  $\mathbf{D}$ , i.e., the displacement vector of this point within an infinite small interval. Then Eq.(2) can be denoted with a Hamilton-Jacobi-like equation as

$$\frac{\partial \varphi}{\partial t} + D_n(\kappa) |\nabla \varphi| = 0 \quad (3)$$

where  $\kappa$  is the curvature of the surface,  $D_n(\kappa)$  is the curvature  $\kappa$  dependent normal velocity of detonation front that can be obtained by theoretical calculations or by experiments. In 3D Cartesian coordinates, the curvature can be deduced as

$$\kappa = \frac{\varphi_x^2(\varphi_{yy} + \varphi_{zz}) + \varphi_y^2(\varphi_{xx} + \varphi_{zz}) + \varphi_z^2(\varphi_{xx} + \varphi_{yy})}{2(\varphi_x^2 + \varphi_y^2 + \varphi_z^2)^{3/2}} - \frac{(\varphi_x \varphi_y \varphi_{xy} + \varphi_x \varphi_z \varphi_{xz} + \varphi_y \varphi_z \varphi_{yz})}{(\varphi_x^2 + \varphi_y^2 + \varphi_z^2)^{3/2}}$$

Usually the  $D_n(\kappa)$  has the following form of function,

$$D_n(x, y, z) = D_n(D_{CJ}, x, y, z; \kappa, R_0, T_0, \dots) \quad (4)$$

Where  $D_{CJ}$  is the ideal steady detonation velocity in a boundless explosive bulk.  $R_0$  is the characteristic size coefficient of the explosive charge, for example the diameter of a cylindrical explosive,  $T_0$  is the ambient temperature. In this paper, a simple linear relation  $D_n = D_{CJ}(1 - \alpha\kappa)$  is utilized, where  $\alpha$  is a constant of the explosive.

The solution of equation (3) under given initial and boundary conditions generates the field  $\varphi(x, y, z, t)$ , and the location of the detonation front at time  $t$  is then simply assigned by a search for the level surface  $\varphi = 0$ .

The initial condition for the level-set equation is the LS function value of each point at  $t=0$ ,  $\varphi(x, y, z; t=0) = 0$  represents the initial detonation front.

The recipe for the boundary condition is same as Aslam and Bdzil's [4]:(1) When the flow in the explosive is supersonic, the continuation boundary is applied. (2) When the flow in the explosive is sonic or subsonic, the sonic condition is applied or the boundary angle must be adjusted until the pressure in the inert and the explosive are equilibrated. See details in [4].

### 3 NUMERICAL CALCULATIONS

#### 3.1 The calculation of ideal detonation

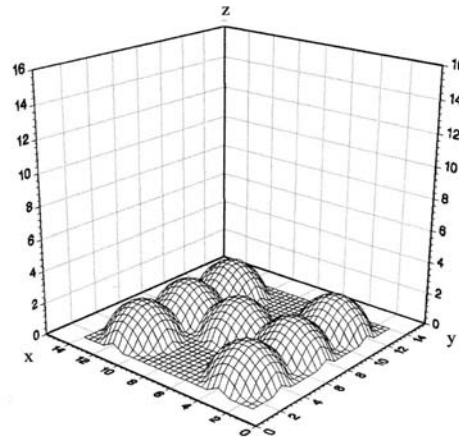
#### propagation

Assumed that the detonation wave propagating at a constant velocity  $D_{CJ}$  in the explosive charge. In this paper, we let  $D_{CJ} = 8.0$  km/s and calculate the detonation front propagating in explosives initiated at seven points or on the center line at the bottom.

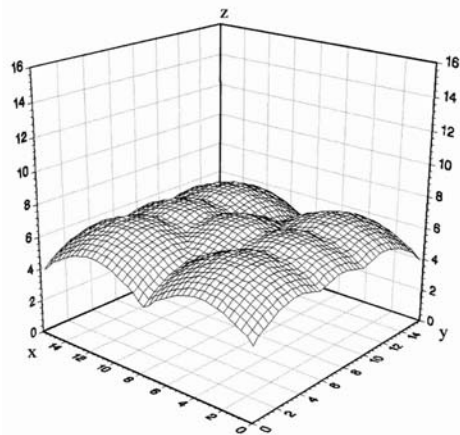
##### 3.1.1 Seven-point initiated detonation wave

The explosive cube with side length 16 mm is initiated at seven points at the bottom whose coordinates are (4,4,0), (4,8,0), (4,12,0), (12,4,0), (12,8,0), (12,12,0) and (8,8,0) respectively. In the calculation the space step and the time step are 0.4 mm and 0.01  $\mu$ s respectively.

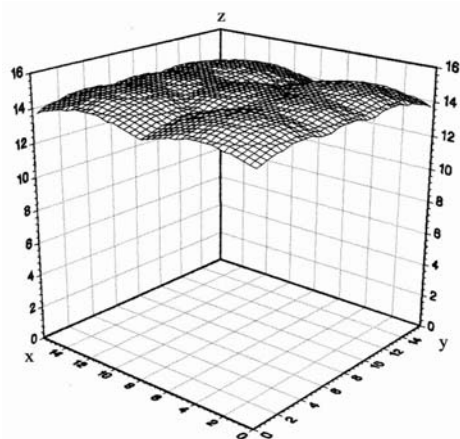
The detonation fronts at different time are shown in Figure 1, the hemisphere like detonation waves appear at first, then they merge with each other and intersect the edges of the cube, and eventually the front burns out of the cube.



(a)  $t=0.30 \mu$ s



(b)  $t=0.90 \mu\text{s}$



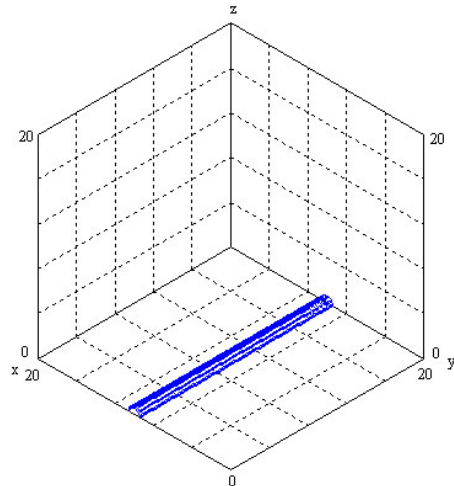
(c)  $t=1.90 \mu\text{s}$

**FIG. 1. SEVEN-POINT INITIATED  
DETONATION FRONT AT  
DIFFERENT TIME**

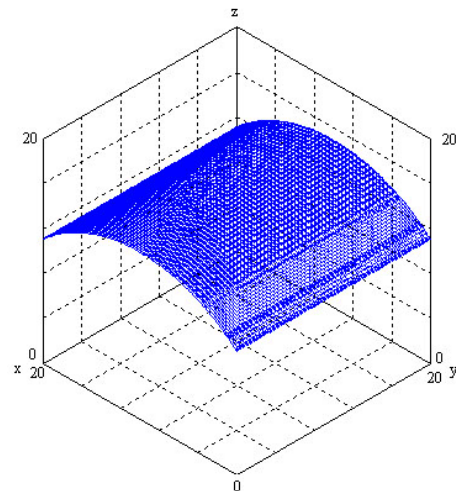
### 3.1.2 Single line initiated detonation wave

A cube explosive with side length 20 mm is initiated on a center line at the bottom, the calculation space step and time step are 0.2 mm and 0.001  $\mu\text{s}$  respectively. The detonation fronts at different time are shown in Figure 2, and

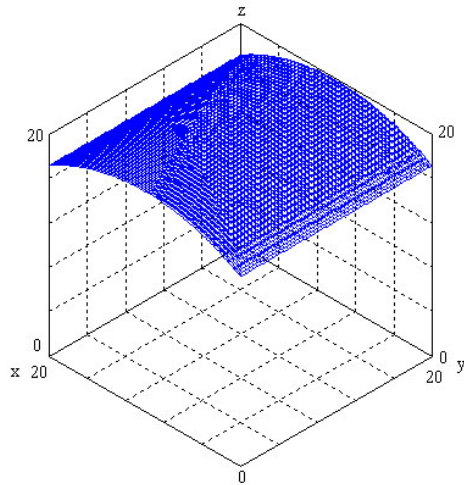
the intersection curves of detonation front with the section plane  $x=10$  at different time are shown in Figure 3 (the time interval between adjacent curves is 0.078  $\mu\text{s}$ ). These figures show that the detonation front shape evolves from cylindrical to planar one.



(a)  $t=0.078 \mu\text{s}$

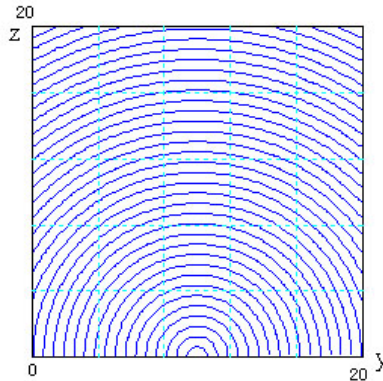


(b)  $t=1.865 \mu\text{s}$



(c)  $t=2.486 \mu\text{s}$

**FIG. 2. SINGLE LINE INITIATED DETONATION AT DIFFERENT TIME**



**FIG.3. THE INTERSECTION CURVES OF DETONATION FRONTS WITH THE SECTION PLANE X=10 AT DIFFERENT TIME ( $\Delta t = 0.078 \mu\text{s}$ )**

### 3.2 Numerical simulations of detonation waves in explosive JB9014 slab and cylinder

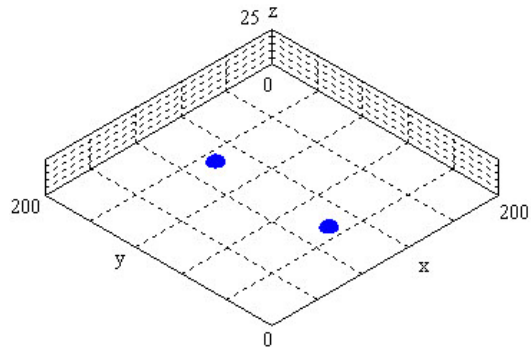
The explosive JB9014 is an

TATB-based insensitive explosive (IHE) with better thermal stability and lower sensitivity to impact. Employing the code LS3D, the detonation propagation in JB9014 slab or cylinder initiated in different way has been numerically simulated; some calculation results are compared with the experimental data. The  $D_n(\kappa)$  relation  $D_n = 7.71(1 - 0.875\kappa)$  (km/s) is used in the calculation.

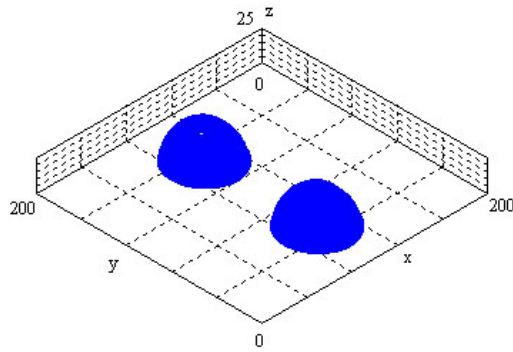
#### 3.2.1 Two-point initiated detonation wave in the JB9014 slab

A JB9014 explosive slab of 200 mm in side length and 25 mm in thickness is initiated at two points whose coordinates are (100,50,0) and (100,150,0). The space step and time step in calculation are 0.5 mm and 0.01  $\mu\text{s}$  respectively.

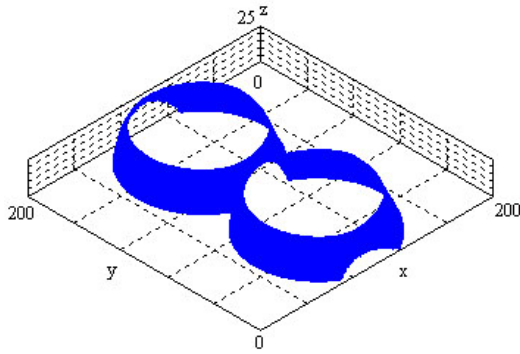
The calculation results of detonation wave at different time are shown in Figure 4. Figure 5 shows the detonation front breaking out on the top line (100,y,25). The calculated waveform (dashed line) agrees well with the experimental observation (solid line) [5].



(a)  $t=0.88 \mu\text{s}$

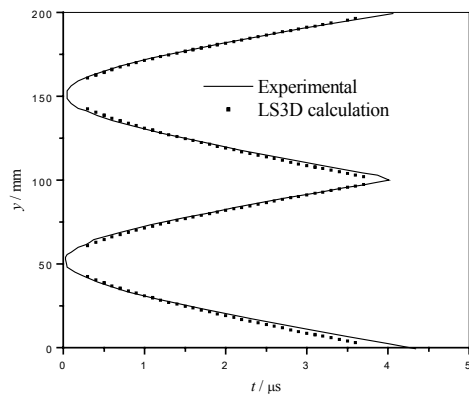


(b)  $t=4.03 \mu\text{s}$



(c)  $t=7.19 \mu\text{s}$

**FIG.4. TWO-POINT DETONATION, DETONATION FRONTS AT DIFFERENT TIME IN JB9014**

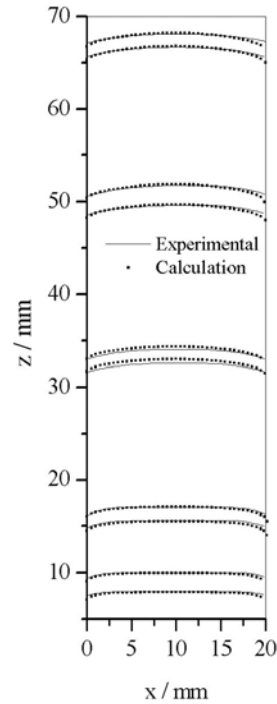


**FIG.5. THE BREAK OUT OF THE DETONATION FRONT ON TOP LINE OF THE EXPLOSIVE SLAB**

### 3.2.2 Detonation propagation in a JB9014 cylinder

A JB9014 explosive cylinder of 20 mm in diameter and 70 mm in length is initiated at the bottom when  $t=0$ . The space step and time step in calculation are 0.5 mm and  $0.005 \mu\text{s}$  respectively.

The calculation results of detonation front (dashed lines) at different time are shown in Figure 6. The time corresponding to the curves upwards are 1.025, 1.295, 2.015, 2.220, 4.315, 4.496, 6.535, 6.820, 8.810 and  $9.000 \mu\text{s}$  respectively. The solid lines are the experimental results [5].



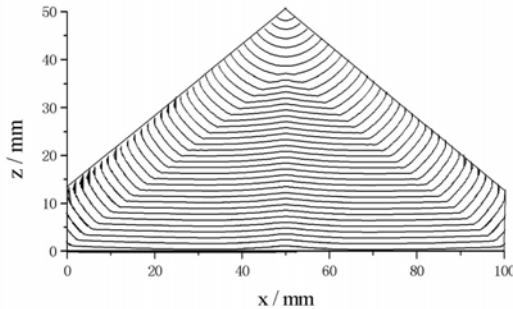
**FIG.6. DETONATION FRONTS AT DIFFERENT TIME IN JB9014 CYLINDER**

### 3.3 Other calculation results

#### 3.3.1 Detonation wave propagation in the plane wave lens

An explosive plane wave lens of 100 mm in diameter and 50 mm in height is combined by two parts of explosives with different detonation velocity. The upper one is Comp. B with detonation velocity 7.8 km/s, and the lower one is Baratol with detonation velocity 4.7 km/s. The propagating process of detonation wave in the lens has been simulated numerically with the code LS3D, the space step and time step are 0.4 mm and 0.002  $\mu$ s respectively.

The detonation fronts are shown in Figure 7, and the evolution from a diverging wave to a planar one can be displayed distinctly.

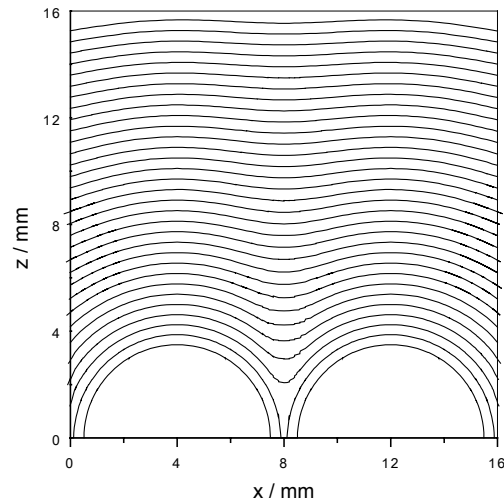


**FIG.7. DETONATION WAVE PROPAGATING IN THE EXPLOSIVE PLANE WAVE LENS ( $\Delta t = 0.256 \mu$ S)**

#### 3.3.2 Interaction of two-point initiated detonation waves

A comp. B explosive cube with side

length 16 mm each is initiated at two point whose coordinates are (4,8,0) and (12,8,0) respectively. Assumed that the radius of both initial spherical detonation waves is 3.5 mm. The propagating process of the two spherical detonation waves in the explosive cube has been simulated numerically. The space step and time step in calculation are 0.2 mm and 0.005  $\mu$ s respectively. The  $D_n(\kappa)$  relation  $D_n = 8.04(1 - 0.16\kappa)$  is used.



**FIG.8 THE INTERACTION OF TWO SPHERICAL DETONATION WAVES ( $\Delta t = 0.256 \mu$ S)**

When two divergent spherical detonation waves collide, regular reflection is first observed, and subsequently, irregular reflection develops. The mach stem is the bridge that develops between the spheres. It is very difficult to simulate the interacting process of two spherical detonation waves in three-dimensional space

numerically. In code LS3D, the interacting process of two spherical detonation waves is simplified as following: when the angle formed by the normal of the two spherical front at the interaction point meet the mach reflection condition, then the propagation velocity of detonation front at the interaction point is adjusted to higher one. Else, no change is made.

The calculation results are shown in Figure 8. The two detonation waves intersect, then merges, and eventually evolves a flat wave gradually.

#### 4 CONCLUSIONS

All the calculated results show that: the code LS3D can be used in calculating the detonation wave propagation in explosive charges initiated in different way. It can deal with different shaped and component multi-medium explosive charges. The validity of calculated results depends on the accurate front curvature dependent detonation velocity and the assignment of edge angle between explosive with air or medium. The measurement of these key parameters is an important study aspect of detonation wave propagation.

It is very difficult and ineffective to simulate the three-dimensional detonation wave propagation by means of common fluid mechanics code especially for that in IHE. But the Level

Set method is a practicable approach. The combination of LS method and the ghost fluid method is promising in engineering calculations of IHE devices.

#### References

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